## **RANDOM VARIABLE**

A rule that assigns a real number to each outcome is called random variable.

The rule is nothing but a function of the variable *X* that assigns a unique value to each outcome of the random experiment.

When a variable *X* takes the value  $x_i$  with he probability  $p_i$  (*i*=1,2,3,...,*n*) then *X* is called **random** variable or stochastic variable or variate.

There are two types of random variable : Discrete Random Variable and Continuous Random Variable.

## **DISCRETE RANDOM VARIABLE**

A random variable *X* which can take only a finite number of values in an interval of the domain called discrete random variable.

Example :

- Number of mistakes in a page.
- Number appearing on the top of a die.

### **DISCRETE PROBABILITY DISTRIBUTION**

If a random variable x can assume a discrete set of values say  $x_1, x_2, ..., x_n$  with respect to probabilities  $p_1, p_2, ..., p_n$  such that  $p_1+p_2+...+p_n=1$  then the occurrences of value  $x_i$  with respective probabilities  $p_i$  is called discrete probability distribution of X.

Example : Consider the experiment of throwing a pair of dice

Let *X* denotes the integer between 2 and 12

X	2	3	4	5	6	7	8	9	10	11	12
P(X)											

Then discrete probability distribution of X with probabilities P(X) is given by

## **Probability Function or Probability Mass Function (pmf)**

Probability Function or Probability Mass Function (pmf) of a random variable X is a function is a function p(x) which gives the probabilities corresponding to different possible discrete set of values say  $x_1, x_2, ..., x_n$  of variable x.

 $p(x_i) = p(x = x_i)$  = Probability that on variable x assumes value  $x_i$ 

The function p(x) satisfies the condition

 $(i)p(x_i) \ge 0$ 

 $(ii)\sum p(x_i)=l$ 

## **Cumulative Distribution Function (Distribution Function)**

If *X* is a random variable then  $P(X \le x)$  is called the cumulative distribution function (cdf) or distribution function and is denoted by F(x).

So, 
$$F(x) = P(X \le x)$$

### **Expectation of a Discrete Random Variable**

If x is a discrete random variable which assumes the discrete set of values  $x_1, x_2, ..., x_n$  with the respective probabilities  $p_1, p_2, ..., p_n$  then the expression or expected value of x is denoted by E(X)and defined as

$$E(X) = p_1 x_1 + p_2 x_2 + \dots + p_n x_n = \sum_{i=1}^n p_i x_i$$

Similarly, the expected value of  $X^2$  is defined as  $E(X^2) = \sum_{i=1}^{n} p_i x_i^2$ 

## **Properties of Expectation**

- 1. If X is a random variable and a be constant then
  - *i.* E(a) = a
  - *ii.* E(aX) = aE(X)

*iii.*  $E(X - \mu) = 0$ 

- 2. If x and y are two random variables then  $E(X \pm Y) = E(X) \pm E(Y)$
- 3. E(XY) = E(X)E(Y) if X and Y are two independent random variables.
- 4. If y = ax + b where *a* and *b* are constants then E(Y) = E(aX + b) = aE(X) + b

A pair of coin is tossed, what is the expected value of getting head?

Let X = number of heads

X = 0, 1, 2

Probability Distribution is given by

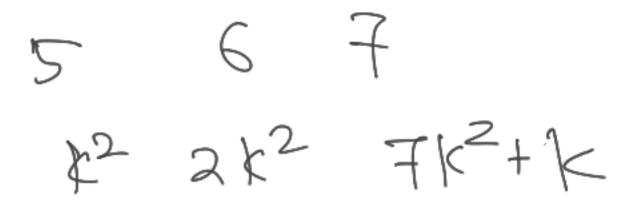
X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$E(X) = (\frac{1}{4} \times 0) + (\frac{1}{2} \times 1) + (\frac{1}{4} \times 2) = 1$$

For the discrete probability distrybution 1 2 3 4 5 6 7 Õ X t o k 2k 2k 3k  $k^2$   $2k^2$   $Fk^2 + k$ 

## Determine

0 K (r) mean (iii) vooriance (1) smallest value of K s.t. P(X ≤ X)>1/2



We know that,

Z f(x) = 1

=)  $0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + K = 0$ 

-3 10 K2+9K-1=0

=) K = - ( 031 /10

K=-1 is not possible

600 K=110

Mean = Zxf(x)

= 3.66

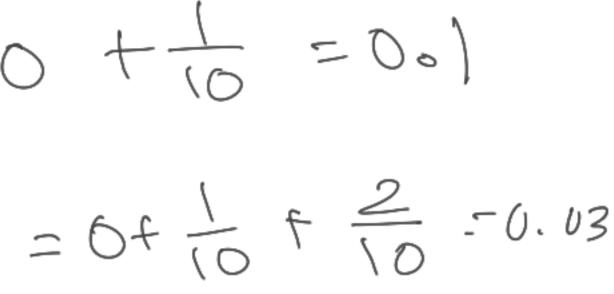
Vociance =  $E[x^2] - (E[x])$ 

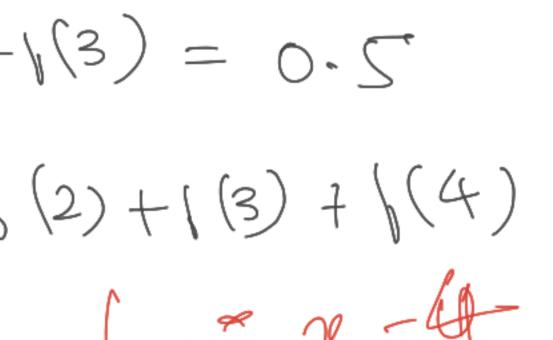
 $= \left[ 0 + \left( 1 \times \frac{1}{10} \right) + \left( 2^2 \times \frac{2}{10} \right) + \left( 3^2 \times \frac{2}{10} \right) + \left( 4^2 \times \frac{3}{10} \right) + \left( 4^2 \times \frac{3}$  $(5^{2} \times \frac{1}{100}) + (6^{2} \times \frac{2}{100}) + (7^{2} \times \frac{7}{100}) - (3 - 6)^{2}$ 

= 37. F

 $\int b(x) = p(x)$ 

 $P(N \le 0) = f(0) = 0$  $P(x \le 1) = f(0) + b(1) = 0 + \frac{1}{10} = 0.0$  $P(X \le 2) = b(0) + b(1) + b(2) = 0 + \frac{1}{10} + \frac{2}{10} = 0.03$  $P(X \leq 3) = f(0) + f(1) + f(2) + f(3) = 0.5$  $P(X \leq 4) = f(0) + f(1) + f(2) + f(3) + f(4)$ = 0.8 ( $\frac{0}{2}x = 4$ " Smallest volue of x s.f. P(X = N)> = 134

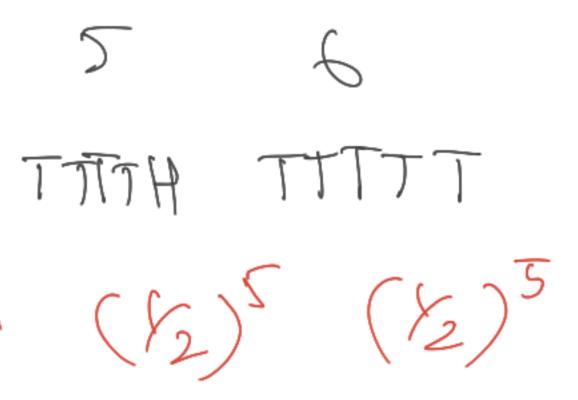




A faise coin is tossed until head on five fails occurs. Find expected no. of tosses of the coin. W = NO, OF + OSSPSX can take values 1,2,3,4,5,6

X (234556 TTH TITH TITH TITT H TH Outcome P(X)  $K_{2}(K_{2})^{2} (K_{2})^{3} (K_{2})^{4} (K_{2})^{5} (K_{2})^{5}$ 

 $\hat{\rho}$ ,  $E[X] = \Sigma \chi p(x) = (^{9}6)$ > Expected no. of tosses ~2

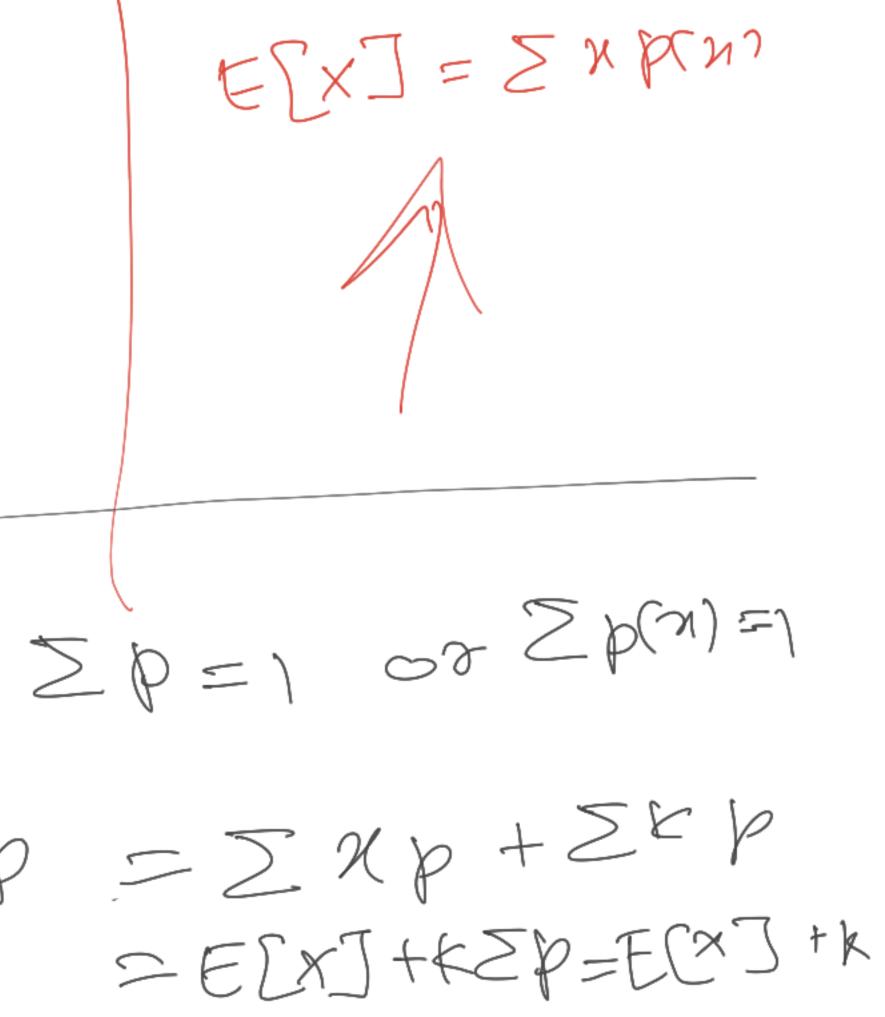




If X and Y are discrete random variables and k is a constant then prove that () E[X+k] = E[X] +k  $\bigcirc E[X+Y] = E[X] + E[Y]$ X 4 Pare dèscrete RoVo und & is constant.



 $F[X] = \sum_{i=1}^{n} \chi_{i} p_{i}(\chi_{i})$  $-\sum_{i=1}^{n} p_i = 1$ E[X] = ZNP E[X+K] = Z(X+K)p = ZXp + ZEp



E[X+Y] = E[X+Y]p

=Zxp +ZJp

# ZECXJ + E[Y]



A random vouiables X has the following probability distribution where k is some number  $P(x) = \begin{cases} x , x=0 \\ 2x , x=1 \\ 3x , x=2 \end{cases}$ o phercoise a) Determine value of K THA  $P(x < 2), P(x \leq 2), P(x > 2)$ 

Independent Random Voorable [E[xy] = E[n] E[y] E[XY] = E[X]E[Y] where x fy and independent R.V.

Covariance

If n 47 are two R.V. with mean I & J respectively then covariance

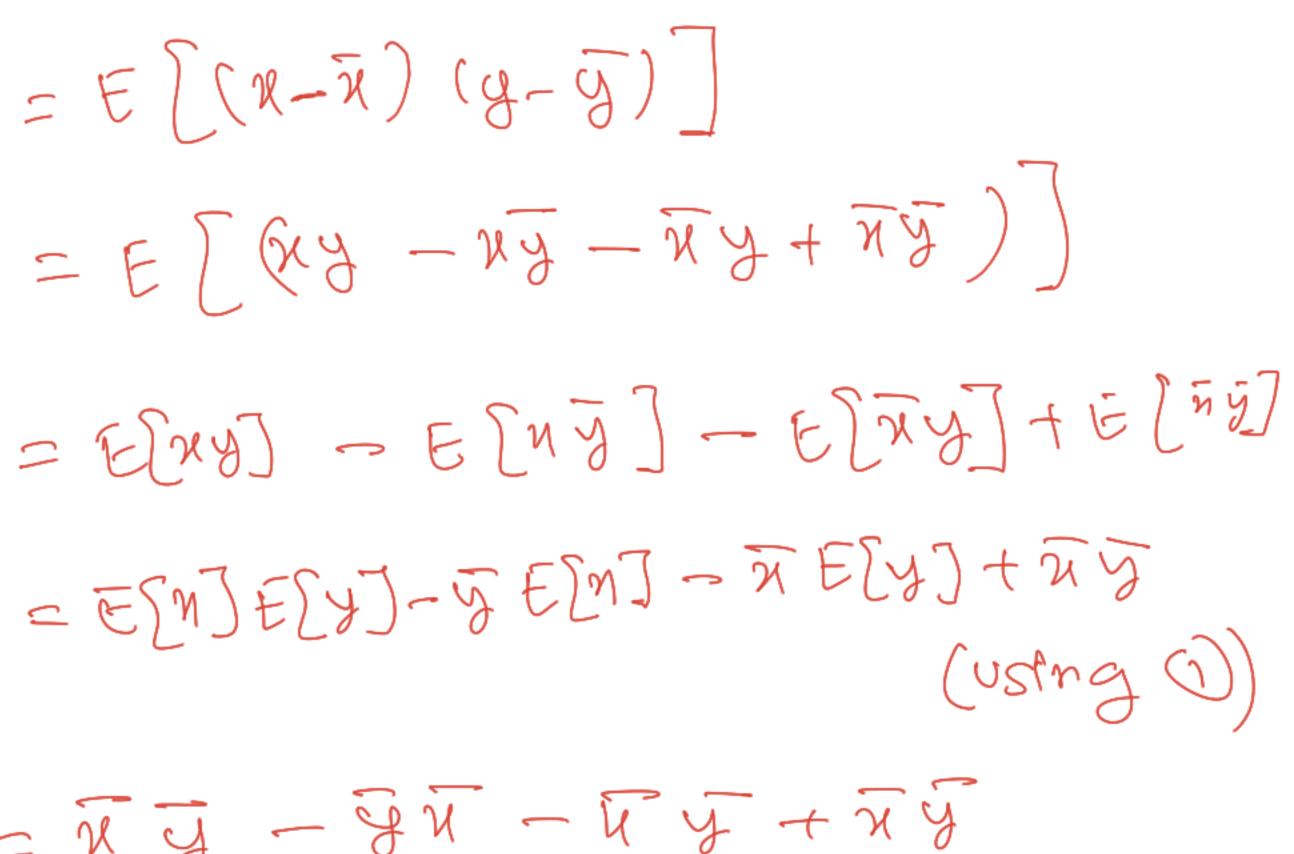
x & y is defined as between

COV(x,y) = E[(x-n)(y-y)]

The coraciance of two independent reandon voriable is zero. let refy be two independent Rov. i = E[ny] = E[n] E[y] - (n)

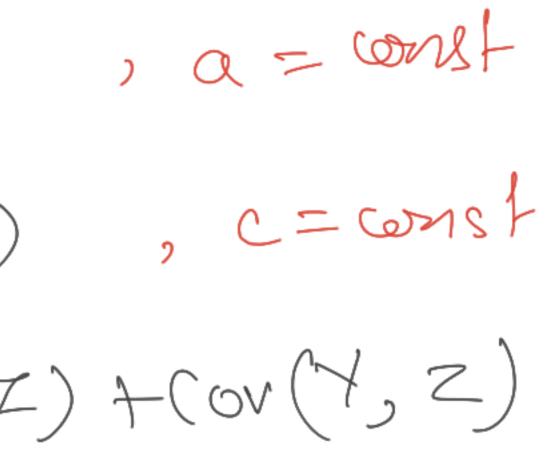


 $Cov(x,y) = E\left[(x-\overline{x})(y-\overline{y})\right]$  $= E \left[ \left( xy - xy - \overline{xy} + \overline{xy} \right) \right]$  $\varepsilon(n+y]$ ニモ「町モビタータモ「の」ーズモビタノナズタ - ECMJ+ELYJ ニズヌージェーインチャンダ x, y const  $\sim$ 



Cosselation Coefficient P = - Cov(x,y)Nov (X) Nov (Y) Properties of covariance ( OV(X,X) = VOX(X)E) if X 4 y are two independent R.V. (m(x, Y)=0

(3) Cov(X, Y) = Cov(Y, X)(A) Gov(aX, Y) = a COV(X, Y)(5) (OV(X+C, N) = (OV(X, Y))(G) (OV(X+Y,Z) = COV(X,Z) + (OV(Y,Z))Properties of coordation  $O - (\leq P(X_CY) \leq )$ 



(2) If P(X,Y) = 1 then Y = aX + b where a>1 3 if P(X,Y) = -1 then Y = aX + b where

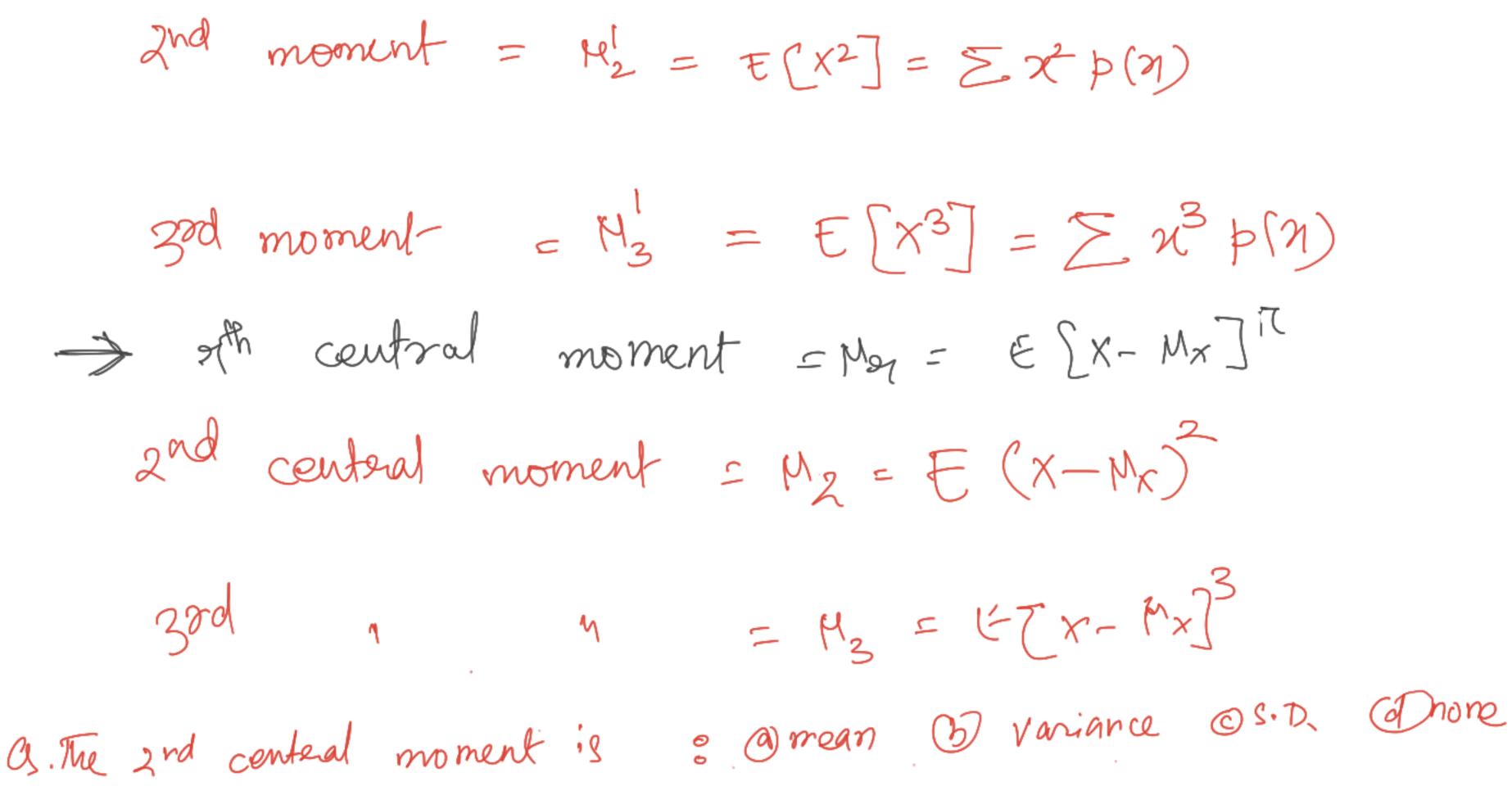
 $\Phi \quad \mathcal{P}(aX+b, cY+d) = \quad \mathcal{P}(X,Y) \quad box \quad a, (7)$ 

aLD

ariable (or its distribution)

on rulated f

 $M_{q} = E(X^{q})$ =  $\sum \chi^{R} P(X = \chi)$  $= \sum \chi^{\pi} p(\chi)$ 



The rith cantral moment of X is 
$$H_{T} = E$$
  
3. Let X be a discrete random variable is  
mass by  
 $P_X(X) = \begin{cases} k_2 & , N=1 \\ k_3 & , N=2 \\ k_6 & , N=3 \\ 0 & , 0 \text{ therefore} \end{cases}$ 
  
Find 3rd moment of X

 $\mathcal{M}_{\mathcal{H}} = E(X - \mathcal{M}_{X})^{\mathcal{H}}$ 

able having probability

## 3rd moment is given by $H'_3 = E[x^3]$

 $= \sum n^3 p(n)$  $= \frac{1}{2}(1)^{3} \times (\frac{1}{3}) + \frac{1}{2}(2)^{3} \times \frac{1}{3} + \frac{1}{2}(2)^{3}$ 23×6)

= 2<sup>3</sup>/3

let X de a discrute R.V. with p.m.f.  $P_{\chi}(\chi) = \begin{cases} 3/4 , \chi = 1 \\ 1/4 , \chi = 2 \\ 0 , otherwise \end{cases}$ Find the 3rd central moment of X. 3rd central moment à griven by M3 = E[X-Mx]  $P_{x} = E(x] = \sum xp(x) = (1 \times \frac{2}{4}) + (2 \times \frac{1}{4}) + 0$ Now,

= = = = = =

 $M_3 = E[X - \frac{5}{4}]^3 = \sum (x - \frac{5}{4})^3 f(x)$ 

 $= (1 - \frac{5}{4})^{3} (\frac{3}{4}) + (2 - \frac{5}{4})(\frac{1}{4})$ 

\*

= 32

.

Moment brewrating Function  
The moment generating 
$$b^n$$
 (m.g.t.) of  
the probability  $b^n$   $b(n)$  is given by  
 $M_n(H) = E(e^{t})$   
 $= \sum_{x} e^{tx} b(n) = \int e^{tx} b(n) dn \in L$   
Here  $t$  is a

t a R.V. X having

\_\_\_\_ Discrute R.V.

Cont. R.V.

real const.

$$(M_{\chi} t_{1}) = E(e^{t\chi}) = E\left[1 + t\chi + \frac{(t\chi)}{2!}\right]$$

$$= E\left[1 + t\chi + \frac{t^{2}\chi^{2}}{2!} + t\chi + \frac{t\chi^{2}}{2!} + t\chi + \frac$$

note ()  $\frac{d^{a_1}}{dt^{a_1}} \left[ (M_X(t)) \right] = N'_a$ 

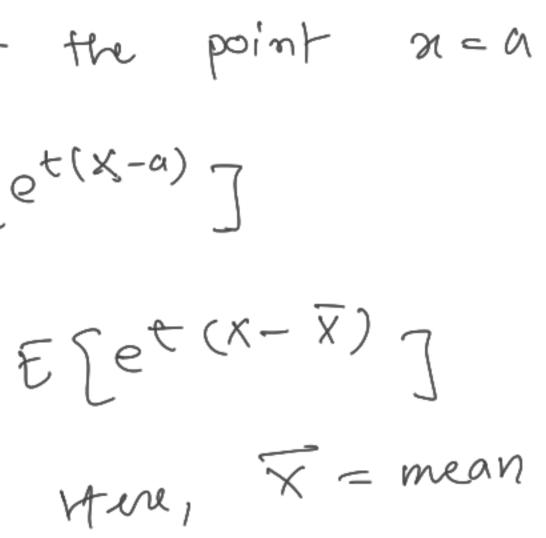
 $\frac{(tx)^{2}}{1+\cdots} + \frac{(tx)^{2}}{31} + \cdots$  $\frac{f}{\pi l} + \frac{f}{\pi l} + \dots$  $2] + \dots + \frac{k\pi}{\pi!} E(x^{m}) + \dots$ sith moment about the origin. 3rd moment, 3r=3  $\frac{d^3}{dt^3}(M_X(t)) = M'_3 \leq 3^{nd}$   $\frac{dt^3}{dt^3}(M_X(t)) = M'_3 \leq 3^{nd}$ 

Moment generating 
$$t^{m}$$
 of  $X$  about  
 $M_{X}(t)$  (about  $n=a$ ) =  $E \int e^{t}$   
 $M_{X}(t)$  (about  $man$ ) =  $E$ 

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Properties  

$$\longrightarrow_{cx}(t) = M_{x}(ct)$$



\*

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. . . . Mx (t)

## le whose moments are

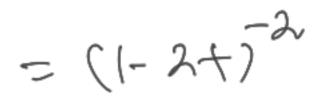
(n+1)[ 21e

01+1). 7! 2"

 $(+1) 2^{TL} = \sum_{n=0}^{40} (n+1) (n+1)^{n}$ 

$$M_{\chi}(t) = \left[ (0f1)(2f)^{\circ} \right] + \left[ (1f1)(2f)^{\circ} \right] + \left[ (1f1)($$

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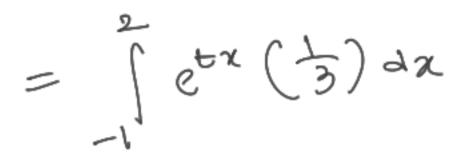
2+)'] + [(2+1)(2+)] + ...

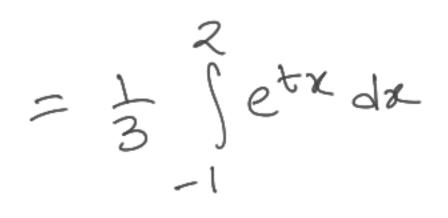
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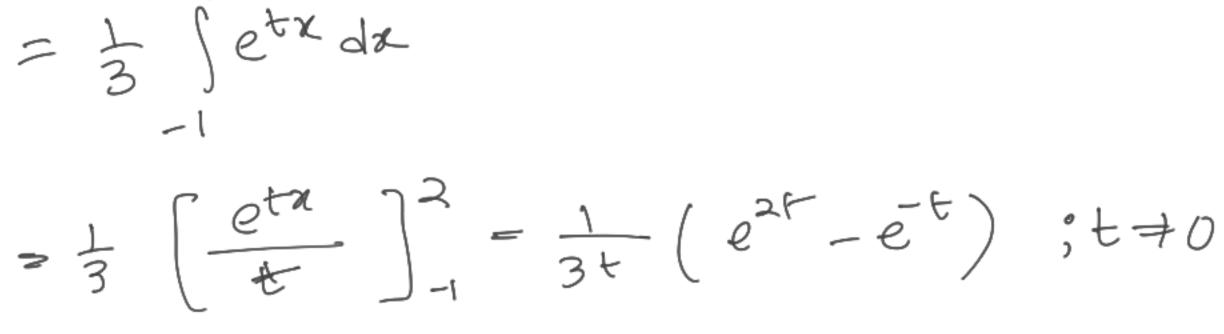
9. Show that mgt of a R.V. X having the probability density K K(x) = { y3, -1<x<2 b(x) = { 0, elsewhere  $M_{\chi}(t) = \begin{cases} \frac{e^{2t} - e^{t}}{3t}, t \neq 0 \\ 1, t = 0 \end{cases}$ r S

 $M_{\chi}(t) = E(e^{t\chi})$ 

= Jetz Zix)dx









Sub. t=0 in () we get  $M_{\chi}(t) = \int e^{0.\chi}(\frac{1}{3}) d\chi$ = = j = j dx = = [x]2 = = [2 - (-1)] = 3 = 1

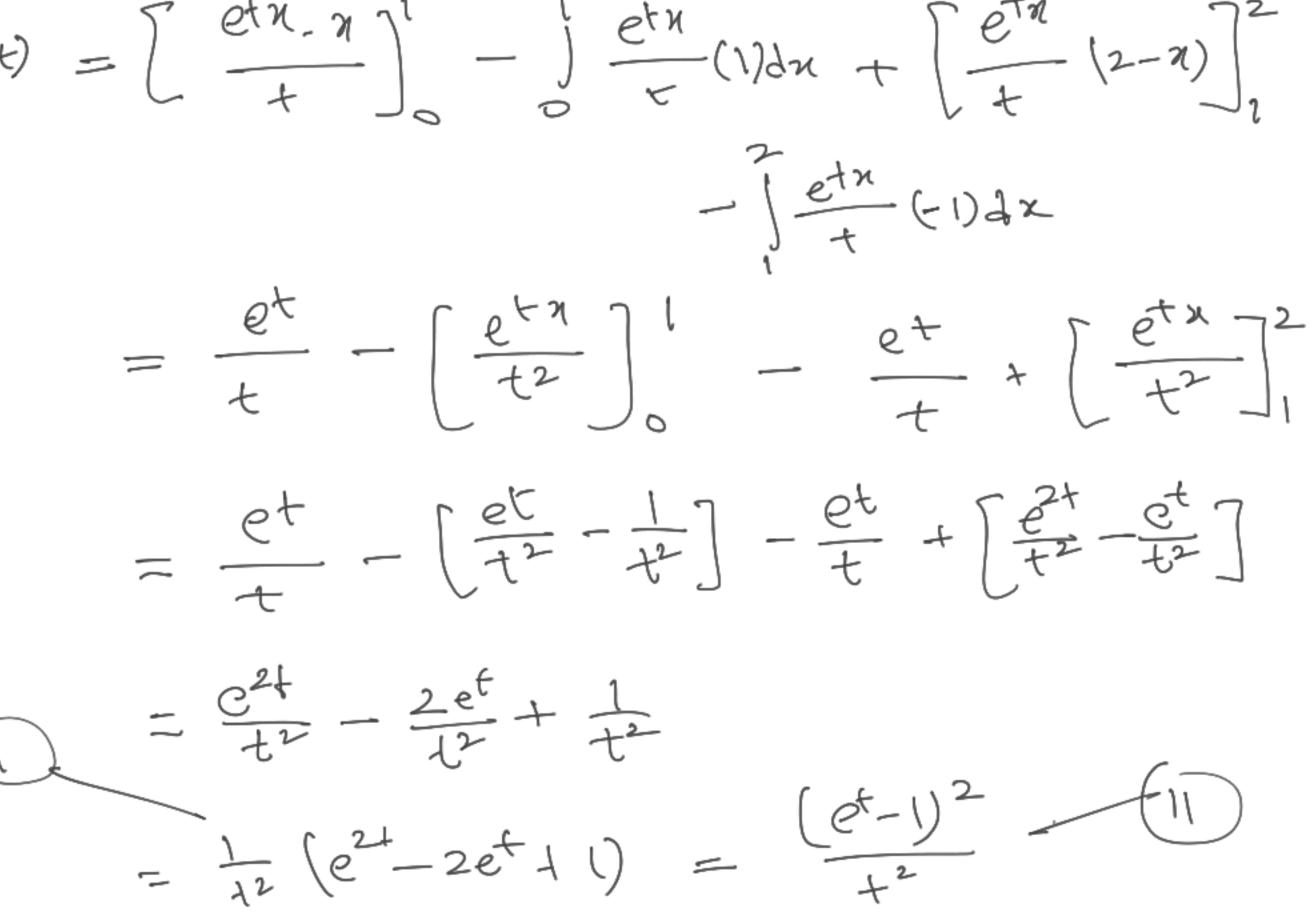
 $\binom{2}{3} M_{X}(t) = \binom{2}{3} \frac{e^{2t} - e^{t}}{3t}, t \neq 0$ 

9. Find the mgt of the R.V. X having the probability density f  $f(x) = \begin{cases} x , 0 \le x < 1 \\ 2 - x , 1 \le x < 2 \\ 0 , otherwise$ 

Also find the mean and variance of X using mgt.

M,(+) = E[etx] = jetx flada + jetx flada

$$M_{X}(t) = \begin{bmatrix} e^{t}x_{-x} \\ -t \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} e^{t}$$



Expanding MxH) in O we get  $M_{x}(b) = \frac{1}{2!} \left[ (1+a+1) + \frac{(a+1)^{2}}{2!} + \frac{(a+1)^{2}}{3!} + \cdots \right] - 2(1+a+\frac{1}{2!})$  $\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right) = 1 + t + \frac{1}{12} + \frac{1}{2} + \cdots$ Mean,  $M'_{i} = coefficient of t in M_{X}(t)$ 

(11) (400 (1))

 $M'_2 = \text{coefficient} \quad of \quad \frac{t^2}{2!} \text{ in } M_x(t) = \frac{7}{12} \times 2! = \frac{7}{6}$ 

Variance  $[M_2] = M'_2 - (M'_1)^2 = \frac{1}{6} - (m^2) = \frac{1}{6}$ 

