Complete Soln or Complete Integral

The soln $f(x,y,z,a,b) = 0$ (1) of a 1st order P.D.E. which contains two arbitrary constants is called complete soln or complete integral.

$b = \phi(a)$ ('b' is a fn of 'a') in (1)

$f(x,y,z,a,\phi(a)) = 0 \leftarrow$ General soln

Particular Sol^n or Particular integral

A sol^n obtained from complete integral by giving particular values to the arbitrary constants is called particular sol^n or particular integral.

Boundary condit^n	Initial condit^n
BVP	IVP

Verify that $e^{-n^2 t} \sin nx$ is a soln of the heat eqn $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$. Hence show that $\sum_{n=1}^{N} c_n e^{-n^2 t} \sin nx$, where $c_1, c_2, \dots, c_N$ are arbitrary const., is a soln of this eqn satisfying the boundary conditn $u(0,t) = 0$ and $u(\pi, t) = 0$

$$u = e^{-n^2 t} \sin nx$$

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(e^{-n^2 t} \sin nx\right) = \sin nx \frac{\partial}{\partial t}\left(e^{-n^2 t}\right)$$

$$= \sin nx \left(-n^2 e^{-n^2 t}\right)$$

$$= -n^2 \sin nx \, e^{-n^2 t} \quad \text{——} \quad (1)$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(e^{-n^2 t} \sin nx\right) = e^{-n^2 t} \frac{\partial}{\partial x}(\sin nx)$$

$$= n e^{-n^2 t} \cos nx$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(n e^{-n^2 t} \cos nx\right)$$

$$= n e^{-n^2 t} \frac{\partial}{\partial x} \cos nx$$

$$= -n e^{-n^2 t} \sin nx \,(n)$$

$$= -n^2 e^{-n^2 t} \sin nx \longrightarrow \text{(ii)}$$

From (i) & (ii) we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Again,

$$u(x,t) = \sum_{n=1}^{N} c_n e^{-n^2 t} \sin nx$$

$$u(0,t) = \sum_{n=1}^{N} c_n e^{-n^2 t} \sin n(0)$$

$$= \sum_{n=1}^{N} c_n e^{-n^2 t} \sin(0)$$

$$= \sum_{n=1}^{N} c_n e^{-n^2 t} (0)$$

$$= 0$$

$$u(\pi, t) = \sum_{n=1}^{N} c_n e^{-n^2 t} \sin \pi n \quad , \quad n = 1, 2, \dots N$$

$$n \in \mathbb{Z}$$

$$= \sum_{n=1}^{N} c_n e^{-n^2 t} \, (0)$$

$$= 0$$

Solve $\dfrac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that $\dfrac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\pi/2$

$$\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y \quad \text{--- (1)}$$

Integrating (1) wrt x keeping y const, we have

$$\frac{\partial z}{\partial y} = -\cos x \sin y + f(y) \quad \text{--- (II)}$$

When $x=0$, $\frac{\partial z}{\partial y} = -2\sin y$

$\Rightarrow -\cos(0)\sin y + f(y) = -2\sin y$ (using (II))

$\Rightarrow -\sin y + f(y) = -2\sin y \Rightarrow f(y) = -\sin y$ --- (III) using (iii) in (ii) we get

$$\frac{\partial z}{\partial y} = -\cos x \sin y - \sin y$$

Integrating both sides wrt y keeping x const, we get

$$z = -\cos x(-\cos y) - (-\cos y) + \phi(x)$$

$$= \cos x \cos y + \cos y + \phi(x)$$

when y is odd multiple of $\pi/2$, $z = 0$

$$0 = 0 + 0 + \phi(x) \Rightarrow \phi(x) = 0$$

$$\therefore Z = \cos x \cos y + \cos y$$

which is the req^d particular soln